

THE AEROSPACE CORPORATION

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SUBJECT: Finite Inertia Corrections
to the Lewis Model Wind Response

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- References:
- (1) Lewis, J. V., "The Effect of Wind and Rotation of the Earth on Unguided Rockets", Ballistic Research Laboratories Report No. 685, Aberdeen, March 1949.
 - (2) Hoult, C. P., "A Simplified Ballistics Model for Sounding Rockets", Proceedings of The Unguided Rocket Ballistics Conference, University of Texas at El Paso - Texas Western College, El Paso, 30 August - 15 September 1966, pp. 9-50.
 - (3) Hoult, C. P., "Launcher Length for Sounding - Rocket Point-Mass Trajectory Simulations", Journal of Spacecraft and Rockets, Vol. 13, No. 12, Dec. 1976, pp. 760-761.
 - (4) Abramowitz, M., and Stegun, I. A., "Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables," National Bureau of Standards Applied Mathematics Series No. 55, 1964, Chapter 7, "Error Function and Fresnel Integrals."

1. INTRODUCTION

One of the commoner tasks occurring in the field of trajectory analysis is calculation of the effects of winds. This is an especially important problem for unguided, fin-stabilized rockets.

In order to minimize computer time and input data preparation, the analyst seeks the simplest model which will do the job. For the wind-trajectory interaction problem, this might be a point mass trajectory model with Lewis theory wind response. This IOC generates the corrections to classical Lewis theory required if the simulation is to be valid everywhere except near the launcher.

2. ANALYSIS

This is a matched asymptotic expansion problem. The outer solution, often referred to as Lewis' Theory, has been known for a long time. As described in ref. (1), the basic idea in Lewis' Theory is that an unguided, fin-stabilized rocket will head instantly into the relative wind. However, at launch, the dynamic pressure, and hence the aerodynamic "spring rate" both vanish while the pitch moment of inertia remains finite. This suggests a Fresnel integral type of inner solution like that in ref. (2). Because the inner solution models in detail the pitch dynamics, its characteristic eq. will be of higher degree than that for the outer solution. An entirely analogous problem for the gravity turn has been solved in ref. (3).

Also, as shown in ref. (2), because winds are fixed relative to the earth, the analysis can be done on a planar basis with rocket roll rate ignored. This is a linearized analysis with infinitesimal wind speed V . The most convenient way to do this problem is to find the step response for a constant V , differentiate to obtain the impulse response, and then find the solution for arbitrary V by superposition.

3. INNER SOLUTION

This follows the notation and procedures of ref. (3). The body axis equations of motion are

$$\dot{w} - \dot{\theta} u = 0 \quad (1)$$

$$I\ddot{\theta} = M_\alpha \left(\alpha + \frac{V}{u} \right) , \quad (2)$$

$$\alpha = \frac{w}{u} , \quad (3)$$

$$M_\alpha = -I\lambda^2 u^2, \text{ and} \quad (4)$$

$$\gamma = \theta - \alpha .$$

Here $u, w = x$ and z body axis velocity components,

$\gamma =$ flight path angle,

$I =$ pitch moment of inertia

$M_\alpha =$ aerodynamic spring rate,

$\lambda =$ initial pitch/yaw wave number,

$\alpha =$ angle of attack

$\theta =$ inertial pitch angle, and

$V =$ wind speed in the unperturbed z direction.

It is presumed that the x axis is taken along the axis of symmetry, and that the inertia ellipsoid is a flat disk. The lateral forces are all neglected in eq. (1), which implies that lateral drift is negligible compared with lateral displacement due to repointing. Aerodynamic and propulsive damping torques have been neglected in eq. (2) in order to obtain a simplified solution.

The two dynamical equations can now be summarized as

$$\dot{w} - \dot{\theta}u = 0, \text{ and} \quad (1)$$

$$\ddot{\theta} = -\lambda^2 u (w + V) \quad (6)$$

The next step is to change the independent variable from time to distance along the flight path, s .

$$\frac{d}{dt} = \frac{d}{ds} \frac{ds}{dt} = u \frac{d}{ds} = u ()' \quad (7)$$

In eq. (7) the prime is used to denote the derivative with respect to distance along the flight path. The result of this variable change is

$$w' - \dot{\theta} = 0, \text{ and} \quad (8)$$

$$\dot{\theta}' = -\lambda^2 (w + V) \quad (9)$$

Eliminating the pitch rate $\dot{\theta}$ from these two gives

$$w'' + \lambda^2 w = -\lambda^2 V. \quad (10)$$

For our step wind profile, the general solution to eq. (10) is

$$w = -V + C_1 \sin \lambda s + C_2 \cos \lambda s, \quad (11)$$

where C_1 and C_2 are constants of integration to be found from the initial conditions.

While on the launcher, the rocket is constrained such that both w and $\dot{\theta}$ are zero. When it has travelled to the end of a launcher of length L , these constraints are the desired initial conditions for the wind response problem. That is,

$$w(L) = 0, \text{ and} \quad (12)$$

$$\dot{\theta}(L) = w'(L) = 0 \quad (13)$$

Applying eq's. (12) and (13) to eq. (11) results in

$$w = -V + V \sin \lambda L \sin \lambda s + V \cos \lambda L \cos \lambda s, \text{ or}$$

$$w = -V [1 - \cos \lambda(s-L)]. \quad (14)$$

From eq. (3), the angle of attack is

$$\alpha = -\frac{V}{u} [1 - \cos \lambda(s-L)]. \quad (15)$$

The pitch angle, and the flight path angle take a little more work.

First

$$\dot{\theta} = -\lambda V \sin \lambda(s-L), \quad (16)$$

$$\text{and } \theta = \int \dot{\theta} dt = \int_L^s \dot{\theta} \frac{ds}{u} = -\lambda V \int_L^s \frac{\sin \lambda(s-L)}{u} ds \quad (17)$$

The simplest description of the longitudinal velocity history is that resulting from a constant axial acceleration a . That is,

$$u = \sqrt{2as}. \quad (18)$$

When this is substituted into eq. (17), and the result rearranged, we obtain the form,

$$\theta = V \sqrt{\frac{\pi \lambda}{a}} \left[\sin \lambda L \int_{\lambda L}^{\lambda s} \frac{\cos \lambda s d(\lambda s)}{\sqrt{2\pi \lambda s}} - \cos \lambda L \int_{\lambda L}^{\lambda s} \frac{\sin \lambda s d(\lambda s)}{\sqrt{2\pi \lambda s}} \right] \quad (19)$$

The integrals appearing in eq. (19) are the Fresnel integrals described in Ref. (4). They are defined by

$$C(X) \equiv \int_0^X \frac{\cos t}{\sqrt{2\pi t}} dt, \text{ and} \quad (20)$$

$$S(X) \equiv \int_0^X \frac{\sin t}{\sqrt{2\pi t}} dt \quad (21)$$

Then

$$\theta = V \sqrt{\frac{\pi \lambda}{a}} \left[\sin \lambda L (C(\lambda s) - C(\lambda L)) - \cos \lambda L (S(\lambda s) - S(\lambda L)) \right], \quad (22)$$

and finally, from eq's (5) and (15),

$$\begin{aligned} \gamma &= V \sqrt{\frac{\pi \lambda}{a}} \left[\sin \lambda L (C(\lambda s) - C(\lambda L)) - \cos \lambda L (S(\lambda s) - S(\lambda L)) \right] \\ &+ \frac{V}{\sqrt{2as}} \left[1 - \cos \lambda (s - L) \right]. \end{aligned} \quad (23)$$

If we instead wanted the flight path angle change resulting from a wind impulse of magnitude V_I occurring at an altitude L from liftoff, it is only necessary to find $-\partial \gamma / \partial L$ from eq. (23). Then the impulse response is

$$\gamma = -V_I \sqrt{\frac{\pi \lambda^3}{a}} \left[\sin \lambda L \left(S(\lambda s) - S(\lambda L) \right) + \cos \lambda L \left(C(\lambda s) - C(\lambda L) \right) \right] \\ + \frac{\lambda}{\sqrt{2as}} \sin \lambda (s - L) \quad (24)$$

Now, we know that most of the wind response occurs at very low altitudes near the launcher. The effects of wind on the gross trajectory are found by evaluating

$$\lim_{s \rightarrow \infty} \gamma = V_I \sqrt{\frac{\pi \lambda^3}{a}} \left[\sin \lambda L \left(\frac{1}{2} - S(\lambda L) \right) + \cos \lambda L \left(\frac{1}{2} - C(\lambda L) \right) \right]. \quad (25)$$

This is the outer limit of the inner solution. To the outer solution, γ mimics a change in launcher elevation angle setting.

4. OUTER SOLUTION

The outer solution is a point mass solution based on the Lewis assumption that the rocket heads instantly into the relative wind. The intrinsic coordinate equations of motion are

$$m u \dot{\gamma} = -(T - D) \frac{V}{u} + mg \sin \gamma, \text{ and} \quad (26)$$

$$a = \frac{T - D}{m} - g \cos \gamma. \quad (27)$$

Here

T = Thrust,

D = drag,

m = mass of the rocket, and

g = acceleration due to gravity.

Combining eq's. (26) and (27),

$$u \dot{\gamma} = -\frac{V}{u} (a + g \cos \gamma) + g \sin \gamma. \quad (28)$$

Note that the response to $g \sin \gamma$ has already been published in ref. (3). Furthermore, for most unguided rockets

$$a \gg g^*, \quad (29)$$

resulting in

$$\frac{d\gamma}{du} = - \frac{V}{u^2} \quad (30)$$

For constant V (a step response), and constant a, we find that

$$\gamma = \frac{V}{\sqrt{2as}} - \frac{V}{\sqrt{2aL}} \quad (31)$$

since the initial condition must be that $\gamma(L) = 0$. Once more the derivative of the step response is the negative of the impulse response. It is

$$\lim_{s \rightarrow \infty} \gamma = \frac{-V_I}{2L \sqrt{2aL}}, \quad (32)$$

where V_I is the magnitude of the wind impulse at altitude L.

5. MATCHING PROCESS

The simplest way for satisfying the requirement that the outer limit of the inner solution match the inner limit of the outer solution is to solve the outer problem -- it describes the gross trajectory -- with a wind profile $F(\lambda s) V(s)$. That is, Lewis Theory will be used, but the real wind profile $V(s)$ will be reduced for simulation purposes by multiplying by F.

* We can see that γ is proportional to $1/\sqrt{a}$ in eq. (23). If the vehicle design were such that a was not large, the γ change due to winds would become inordinate.

The essence of this problem is to determine the factor F. Fortunately, the ratio formed by eq's. (25) and (32) will do the job nicely. Here, in all its glory, is the appropriate form for F:

$$F = \sqrt{2^3 \pi (\lambda s)^3} \left[\sin \lambda s \left(\frac{1}{2} - S(\lambda s) \right) + \cos \lambda s \left(\frac{1}{2} - C(\lambda s) \right) \right]. \quad (33)$$

We note that F depends only on the initial pitch/yaw wave number, λ , and on the distance travelled from liftoff, s . The formula for F above is universal; i.e., it should apply for all types of unguided, fin-stabilized rockets.

In attached computer printout, the numerical values of F are given. The numerical data are also shown in the attached graph. It looks much like an exponential.

6. CONCLUSION

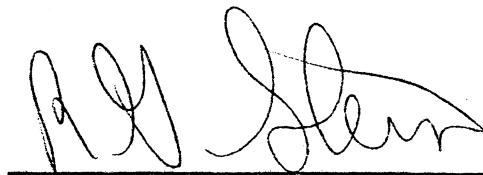
A good, simple trajectory model for an unguided, fin-stabilized rocket flying in a wind field is based on the point mass, Lewis Theory equations of motion. For the simulation to be valid everywhere except near the launcher, the launcher length in the simulation should be found from ref. (3), and the horizontal winds should be multiplied by a factor F given by eq. (33).

Finite Inertia Corrections
to the Lewis Model Wind Response

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102.0

92.0

82.0

72.0

62.0

52.0

42.0

32.0

22.0

12.0

2.0

1.0

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Finite Inertia Correction Factor, F, to
Be Applied to the Lewis Theory Wind
Response as a Function of λs

Notes:

- (1) F should multiply the Lewis theory wind weighting factor to obtain the correct response.
- (2) λ is the liftoff pitch/yaw short period wave number in radians per foot.
- (3) s is the distance traveled along the flight path, in feet, measured from the point of first motion.

 λs

2.2

1.2

0.2

-0.2

-1.2

-2.2

-3.2

-4.2

-5.2

-6.2

-7.2

-8.2

-9.2

-10.2

-11.2

-12.2

-13.2

-14.2

-15.2

-16.2

-17.2

-18.2

-19.2

-20.2

0

0.2

0.4

0.6

0.8

1.0

THIS IS A TABLE OF CORRECTION FACTOR AS A FUNCTION OF DISTANCE ALONG THE FLIGHT PATH. WHEN THE LEWIS THEORY FLIGHT PATH ANGLE CHANGED DUE TO WINDS IS MULTIPLIED BY THE CORRECTION FACTOR, A GOOD APPROXIMATION TO THE REAL DCF DYNAMIC RESPONSE IS OBTAINED.

2. **Digitized by srujanika@gmail.com**

ନାମକରଣ ପରେ ଏହାର ପରିଚୟ ଆଜିର ପରିଚୟ କିମ୍ବା ଏହାର ପରିଚୟ ଆଜିର ପରିଚୟ କିମ୍ବା

27.74% of the total area of the study area is covered by water bodies. The remaining land area is distributed among various land use categories such as agriculture, forest, and settlements.

8
7
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